

Characterizing Mathematically Parameterized Toroidal Propellers for AUV Applications

Haojing Huang^{*1}, Raymond Turrisi^{*1,2}, Ryan Leonard³, Michael Benjamin¹

Abstract—Herein this work, we experimentally characterize parametrically designed toroidal propellers and compare them to conventional propellers which have been designed in OpenProp in terms of static thrust, dynamic thrust, and acoustic performance. Two conventional propellers and seven toroidal propellers each deviating slightly from a nominal toroidal design were tested. For the parametric representation used in this study, propellers designed in OpenProp outperformed toroidal propellers with respect to all thrust metrics considered in this paper. No statistically significant difference was found in their acoustic performance. The goal for this work is to present experimental data comparing optimized conventional propellers designed in OpenProp to a variety of toroidal propellers designed using a particular geometric parameterization.

Index Terms—toroidal propellers; autonomous underwater vehicles; marine propulsion; propeller noise

I. INTRODUCTION

A. Overview

Toroidal propellers have garnered much attention in recent years. This design can generally be described as a ring-shaped, tipless propeller where each blade forms a closed loop. Patents and papers referencing tipless propellers, closed-ring formations, and joined blades, with claims of increased efficiency and thrust while reducing vibration, date back over 130 years [1]–[3]. However, they have been re-popularized by Sharrow Marine since 2010 [4] and MIT Lincoln Lab as of 2017 [5].

Sharrow Marine has demonstrated success and provides data on their propellers compared to conventional boat propeller designs, and claims that their propellers provide greater control authority (tested on azimuth thrusters) in addition to being more efficient, producing less vibration and acoustic noise, and have more reverse thrust than conventional propellers. MIT Lincoln Labs demonstrated a similar propeller design in principle which uniquely complement aerial drones. They also claim to achieve comparable thrust to conventional multi-rotor drone propellers, while reducing acoustic noise as well as any risk of catching, cutting, or snagging on objects in the working environment [5].

Evidently, these advantages lend themselves to Autonomous Underwater Vehicles (AUVs). Designers are concerned with their system efficiency, acoustic signature, and overall acoustic



Fig. 1: The propellers tested in this study, and the mounting fixture used in all experiments.

pollution into an ecosystem - especially in applications such as environmental monitoring [6]. The dominant technology for AUV communication and localization is through acoustics, where ideally the system injects as little noise as possible into the transmissions. Furthermore, small-diameter AUVs have additional challenges where it is ideal to minimize the roll induced by the propeller on the AUV to achieve thrust, such that the total control bandwidth is not spent mitigating a constant roll disturbance.

Unfortunately, the literature on presenting standard design methods or techniques for toroidal propellers is limited. Both standard parametric models and the design tools that use them are steps in the right direction in order to further explore where these propellers can complement the field of marine robotics.

B. Contributions

In this work, we present experimental data on toroidal propellers designed using the parametric description presented in [7]. We present results on:

- Experimental characterization on two propellers developed in OpenProp [8] compared to seven different toroidal propellers designed with the parametric model in [7].
- Static thrust characterization of all nine propellers, tested at ten RPMs each, resulting in over 90 trials.
- Dynamic thrust characterization of seven propellers, across an additional 140 trials.
- Acoustic characterization of three propellers (one regular propeller, two toroidal), across an additional 75 trials.

^{*}Equal contribution, ¹ Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA rturrisi@mit.edu, haojing@mit.edu, ² Woods Hole Oceanographic Institute, Woods Hole, MA 02543, USA, ³ Northeastern University, Boston, MA 02115, USA leonard.ry@northeastern.edu

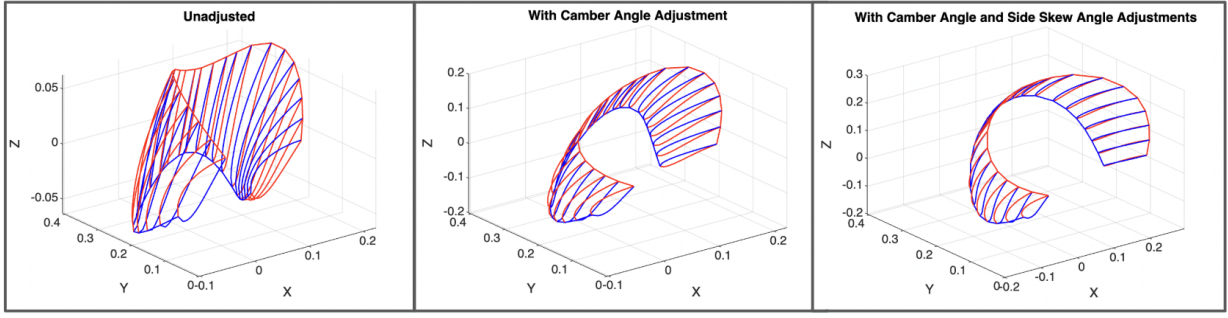


Fig. 2: Propellers generated in MATLAB using the values in Table V, with θ_s and ϕ set to zero (left), only ϕ set to zero (middle), and neither set to zero (right). The blue curves represent the leading edge, while the red curves represent the trailing edge.

II. BACKGROUND

A. Parametrization

Herein we briefly summarize the parametric model of the equations used in the design of these propellers, which was first presented in [7], with an emphasis on implementation. We first define the parameters and variables which characterize the total geometry of a propeller, introduce NACA foil variables, and then the final equations which generate the blades of the propeller.

Table I: Design parameters (from an adjusted table, such as Table V)

Symbol	Description
l	Axial span (position along blade)
r_l	Radial coordinate of reference line
x_l	Rake displacement
b	Chord length
P	Pitch
t	Thickness
f	Camber
θ_s	Skew angle
ϕ	Lateral angle
ψ	Roll angle
α	Vertical angle

Table II: NACA foil variables

Symbol	Description
s	Chordwise position from leading edge
β	Pitch angle: $\arctan(P/(2\pi r_l))$
c_1	Distance from leading edge to generatrix: $c_1 = b/2 - r_l \theta_s / (\cos \beta \cos \alpha)$
$y_b(s), y_f(s)$	Airfoil shape: offset from chord line to back/face (computed from NACA equations using t, f, b)

The process starts with generating a 2D NACA foil at each axis span. To do so, for each row at l , we produce a complete cross section for all values of s between 0 and b , yielding $y_f(s)$ and $y_b(s)$, before passing them through the intermediate calculations and finally equations 6, 7, and 8. The following equations were used to produce a cross section with a standard

4-digit NACA foil. We start with the thickness distribution, where we have

$$y_t(s) = \frac{t}{0.2} \left(0.2969 \sqrt{\frac{s}{b}} - 0.1260 \frac{s}{b} - 0.3516 \left(\frac{s}{b} \right)^2 + 0.2843 \left(\frac{s}{b} \right)^3 - 0.1015 \left(\frac{s}{b} \right)^4 \right) \quad (1)$$

We then define the camber line $y_c(s)$

$$y_c(s) = \frac{4f}{b^2} \cdot s \cdot (b - s) \quad (2)$$

With the thickness distribution and the camber line, we then compose the leading edge and trailing edge of the foil, as $y_f(s)$ and $y_b(s)$ respectively. These symbols are chosen to be consistent with [7].

$$y_f(s) = y_c(s) + y_t(s) \cos(\theta_{\text{local}}) \quad (3)$$

$$y_b(s) = y_c(s) - y_t(s) \cos(\theta_{\text{local}}) \quad (4)$$

Where θ_{local} is defined as

$$\theta_{\text{local}} = \arctan \left(\frac{dy_c}{ds} \right) \quad (5)$$

With the precomputed point for a foil at length l , the exact point along the blade is defined by the following equations.

$$x(l, s) = l + x_l + [(-c_1 + s) \cos \alpha] \sin \beta - [y_i \cos \psi] \cos \beta \quad (6)$$

$$r(l, s) = r_l - \left(-\frac{b}{2} + s \right) \sin \alpha - y_i \sin \psi \quad (7)$$

$$\theta(l, s) = \phi + \frac{1}{r} \{ [(-c_1 + s) \cos \alpha] \cos \beta + [y_i \cos \psi] \sin \beta \} \quad (8)$$

Where y_i refers to y_f and y_b . An example of the resulting propeller blade is depicted in Fig. 2

B. Propeller Thrust Definitions

Propellers are typically characterized with respect to several foundational and widely used equations. We first start by introducing the advance ratio

$$J = \frac{U_p}{nD} \quad (9)$$

The estimated inflow speed at the propeller is defined by $U_p = (1-w)U$, where w is the wake speed, and U is the actual ship speed. Then n is the propeller speed in revolutions per second, and D is the propeller diameter. In our presentation, we set $w = 0$, since the proper experimental apparatus to characterize the wake speed was not available.

With the advance ratio, we then compute the thrust and torque coefficients.

$$K_T = \frac{T}{\rho n^2 D^4} \quad (10)$$

$$K_Q = \frac{Q}{\rho n^2 D^5} \quad (11)$$

Where T is thrust, ρ is fluid density, and Q is the torque at the shaft.

For a thorough discussion on the subject, we refer the reader to [9]. For the results presented in this paper, we deliberately do not present the open water efficiency. In our experiments, we observed that the difference in the theoretical torque with the supplied current and the measured torque at the load cell vary by an order of magnitude, despite recent calibration. With this, we solely calculate K_Q based on the *theoretical* torque, so the relative theoretical torques can be compared with one another, which is directly a function of measured current. The exception is to show the reader our observations the discrepancy in Fig. 7, where for the same propeller, the measured torque is 10x greater than the theoretical torque. To calculate the theoretical torque we take the reported K_v value for the M200 motor as reported by BlueRobotics which is 470 RPM/V, and used the following equations:

$$K_t = \frac{60}{2\pi K_v} \quad (12)$$

$$Q = IK_t \quad (13)$$

Where K_t is the motor torque constant, I is the measured current, and Q is the *theoretical* torque.

The authors would refer the reader to [10], where the published performance metrics on the T200 thruster from BlueRobotics was verified on the same tow tank and load cell. The results, however, report a physically unrealizable efficiency. We believe this is due to sensitivity in the open water efficiency equations and a poor signal to noise ratio. For the reader, we include both the experimental and theoretical K_Q for two propellers in Fig. 7 for reference.

C. Overall Sound Voltage Level

Noise levels are typically characterized using Overall Sound Pressure Level (OASPL) which is calculated by:

$$OASPL = 20 \log_{10} \left(\frac{P_{rms}}{P_{ref}} \right) \quad (14)$$

where P_{rms} is the root mean square of the measured sound pressure level and P_{ref} is the reference pressure level which depends on the fluid in which the sound propagates [11]. However, obtaining absolute pressures requires calibrating the hydrophone with a known sound source such as a pistonphone, which was not available within the scope of this study. Since overall sound is measured relative to a reference pressure, it is still possible to relate the relative sound via the Overall Sound Voltage Level (OASVL), calculated by:

$$OASVL = 20 \log_{10}(V_{rms}) \quad (15)$$

where V_{rms} is the root mean square of the measured voltage signals which is equivalent to the standard deviation of the voltage signal, assuming zero mean. Since the absolute pressure is simply the measured voltage signal scaled by a constant factor, OASPL and OASVL are directly related, through the properties of logarithms, by an additive constant. Therefore, for the purpose of comparing the *relative* noise level between toroidal and standard propellers, comparing the OASVL measurements should be sufficient.

III. METHODS

A. Propeller Design & Fabrication

We first generated a set of conventional propellers using OpenProp in MATLAB [8], in order to present a reference which the toroidal propellers performance can be related to. Two of the 3-blade propellers presented in this paper, referred to as **R1** and **R2**, were designed for the nominal Spurdog AUV [12], targeting a nominal desired operating RPM of 1200 and 800 respectively for a cruising speed of 2 m/s.

To generate the toroidal propellers, we created equivalent Python and MATLAB design tools (each for their own conveniences) that produce 3-dimensional coordinates for individual blade cross sections and two guide rail curves - capturing the leading edge and the trailing edge - with the model described in [7] and discretized values shown in Table V, which is depicted in Fig. 2. These profiles were then imported into CAD, designed around the Spurdog motor hub, and then 3D printed out of Tough 2000 V1 resin on a Form 3 SLA printer from Formlabs. The authors note that alternative resins were also explored (i.e. Rigid 4000 and White), but Tough 2000 gave the best results.

For the propellers presented herein, the tabulated parameter values in Table V formed a baseline toroidal propeller around which a set of parameters was scaled one at a time to produce new distinct toroidal propellers. The scaling specifications used to generate the propellers in this study are shown in Table III below. Please note that the propeller diameter (D) used in this study is 92 mm and the total axis span (L)

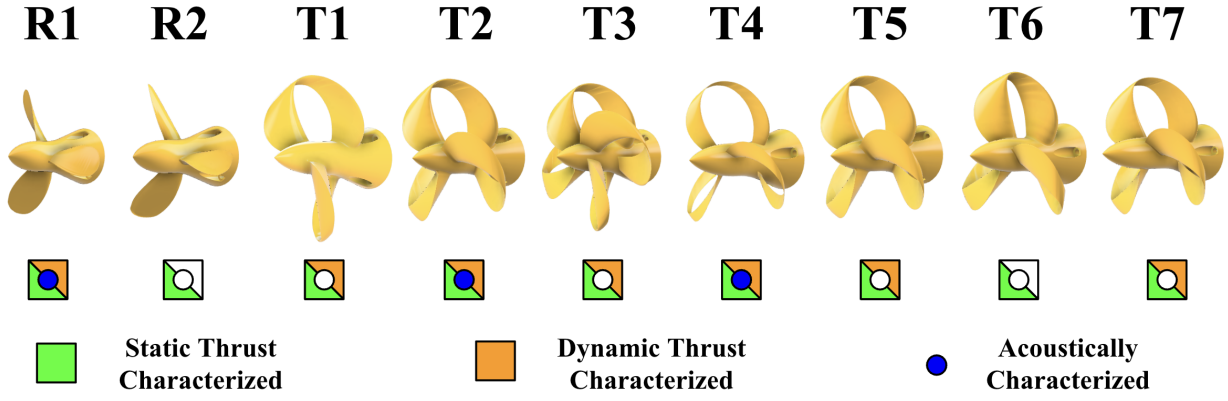


Fig. 3: Renderings of the propellers experimentally characterized in this paper.

used is 20 mm. Renderings and a summary of how each propeller characterized is depicted in Fig. 3, while a photo of the propellers tested along with the experimental apparatus is depicted in Fig. 1.

Table III: Toroidal Propeller Specs

Propellers	n Blades	b/D	t/b	θ_s/rad	α
T1	2	x1	x1	x1	x1
T2	3	x1	x1	x1	x1
T3	4	x1	x1	x1	x1
T4	3	x1/2	x1/2	x1	x1
T5	3	x1	x1	x1	x4/3
T6	3	x1	x1	x1/10	x1
T7	3	x1	x1/2	x1	x1

B. Experimental Setup

1) *General*: To drive the experimental apparatus, a wireless setup with a 14.8V lithium battery was designed, which supported a Raspberry Pi 4, a Teensy 4.1 microcontroller, the Blue Robotics “Basic ESC”, the INA 260 Current and Voltage sensor from Adafruit, and the Hobbywing RPM sensor. A laptop projected a Wi-Fi network, which the Raspberry Pi 4 connected to in order to mobilize the setup between testing sites and reduce the need for tethers on the moving carriage. The operator would SSH into the Raspberry Pi 4 while driving the Teensy 4.1 over serial communication commands. The Teensy 4.1’s native SD card slot logged the current, voltage, and RPM information. Separately, the tow tank was initialized to start recording data while commands were sent to drive the carriage at different advance velocities. The Hobbywing RPM sensor measurements were compared with a Neiko Tachometer across a variety of RPMs in-air to confirm the measurements were accurate.

2) *Thrust Characterization*: All thrust data in this study was taken in a MIT Sea Grant’s tow tank and is modeled after a similar experimental setup demonstrated in [13]. This tow tank is actuated and fitted with a high-resolution ATI-Gamma six-axis force sensor load cell, where both static thrust

can be measured, and dynamic thrust can be measured up to 2 m/s. In our setup, we attach to this load cell through a standard dovetail connection, which connects to a custom aluminum bracket and tube. This tube is 16-inches in length, and shrouded with stacked 3D printed struts. This submerges and provides structure for the M200 brushless DC (BLDC) motor from Blue Robotics to which the propeller is attached to and shrouded in a custom streamlined cowling. On the suspended carriage, an additional electrical stack which allows the M200 motor to be controlled wirelessly while measuring voltage, current, and RPM data. A photo and a schematic of the experimental setup is shown in Fig. 4

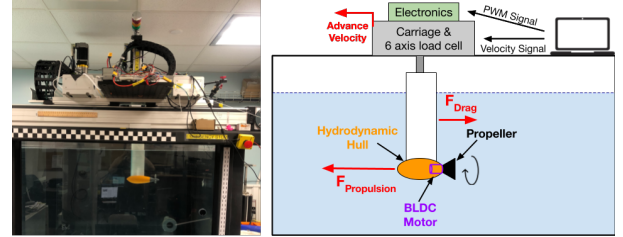


Fig. 4: Thrust characterization experimental setup, where the propeller is attached to the tow tank carriage and force data are measured by the onboard load cell sensor. PWM signals and velocity signals are sent from the computer to the BLDC motor and the carriage to start and stop each test run.

Static thrust data was collected for each propeller driven at 5 different PWM signals in both the forward and reverse directions. Dynamic thrust data was collected for a subset of the propellers at advance velocities of 0.2 m/s, 0.4 m/s, 0.6 m/s, and 0.8 m/s while the motor is driven at the same 5 different PWM signals in the forward direction. Due to limitations with the hardware setup, the PWM values used in this experiment were kept below a duty cycle of 0.375 to prevent damaging the load cell and remain in the valid sensing range of the load cell. Care was taken to verify that the axis of thrust was aligned with the load cell’s x-axis. In post processing by inspecting the load projected onto the x and y axes of the load cell, an average alignment error of

2 degrees was found, which contributes to less than 3% of random error.

3) *Acoustic Characterization*: A similar setup was used to take the acoustic data for this study in a closed-water environment in MIT's Large Towing Tank using the C75 Hydrophone from Cetacean Research. The propeller is submerged in water with the same dovetail and custom aluminum bracket and tube structure. The hydrophone is positioned at the same depth approximately one meter downstream of the propeller to measure the close field noise. A photo and a schematic of the experimental setup is shown in Fig. 5

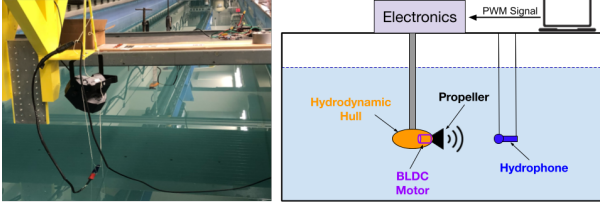


Fig. 5: Acoustic characterization experimental setup, where the propeller is positioned at the center of the tank and the close field noise is measured at 100 kHz by the C75 hydrophone. PWM signals are sent from the computer to the BLDC motor to start and stop each test run.

A subset of the initial set of propellers were selected for acoustic characterization - The best performing conventional propeller R1, the original toroidal propeller T2 as tabulated in Table V, and the thinnest propeller T4. Each propeller was driven statically at 5 different PWM signals consecutively for 2 seconds each and the close field noise was measured at a sampling frequency of 100 kHz. Without the rated limit of the load cell, the PWM range used in this experiment was up to a duty cycle of 0.625. Care was taken to verify that acoustic reflection from the towing tank walls and consecutive sampling from one motor speed to another did not affect the noise measurements for each PWM signal. Due to the limitations of the apparatus, it was difficult to isolate the propeller from the combined noise of the motor and propeller. However, for the purpose of comparing the relative noise level between toroidal and standard propellers, analyzing the noise of the entire system should be sufficient.

IV. RESULTS AND DISCUSSION

A. Thrust Characterization

To begin investigating the thrust performance of each propeller, the raw load cell data, RPM data, and electrical data shown in Fig. 6 were first plotted and time intervals for when the propeller is turned on were manually selected for each trial. A MATLAB script was used to window into each of these intervals and calculate its corresponding average forces, moments, RPM, current, and voltage measurements.

A plot of the thrust-to-power ratio, the thrust coefficient, and the torque coefficient [9] for the initial set of conventional propellers and toroidal propellers are shown in Fig. 8. From these preliminary results, we observe that for static conditions,

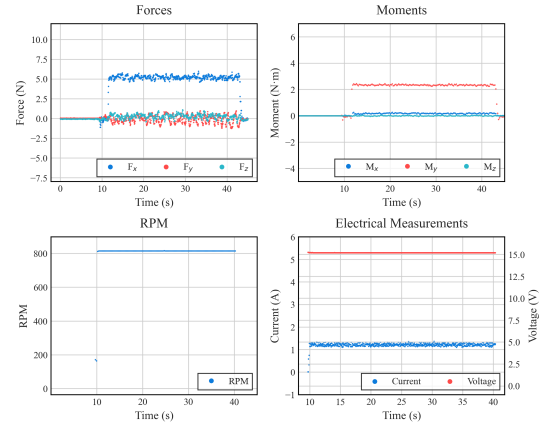


Fig. 6: A plot of raw sensor data from R1 propeller trial where the motor was driven at a duty cycle of 0.25 and the tow tank carriage was driven an advance velocity of 0.2 m/s.

toroidal propellers generally have more symmetry in forward and reverse thrust, lower thrust coefficient, and comparable thrust to power ratio to that of top performing conventional propeller. As a reference, the top performing conventional propeller, labeled **R1**, was designed for 8 N of thrust at 1200 RPM, and experimentally we measured 10.6 N at 1100 RPM. From these results, we also observe that among the toroidal propellers we tested, propeller **T7** is the 'best' among the toroidal propellers, as it has the highest thrust-power efficiency and thrust coefficient.

For static thrust, we present the thrust-to-power ratio, the thrust coefficient, and the torque coefficient, at varying RPMs. **R1** had the largest overall thrust-to-power ratio and thrust coefficient. Comparably, while the torque coefficient from **R1** was slightly larger than all other toroidal propellers tested, it had the greatest overall performance.

For dynamic thrust, we present tabulated results for a linear fit, which was conducted on each propeller tested. For reference, the data from **R1** and **T2** are depicted in Fig. 7, showing both the raw data points and the approximate general linear trends. We created a linear fit for each propeller, and provide the quality of and parameters for each fit in Table IV. With this, it should be noted that **R1** had the greatest overall thrust coefficient and comparable linear decay.

B. Acoustic Characterization

To begin investigating the acoustic performance of each propeller, the raw hydrophone data and RPM data shown in Fig. 9 were first plotted and visually inspected to ensure the expected behavior is observed. Intervals of discrete "jumps" in voltage measurements and RPM measurements, corresponding to increases in the PWM signal, are then manually selected for each trial. A MATLAB script was used to window into each of these intervals and calculate its corresponding average RPM value and OASVL value based on Equation 15.

After obtaining the OASVL and RPM data for each trial, a logarithmic fit with the form $OASVL = A \ln(\Omega) + B$,

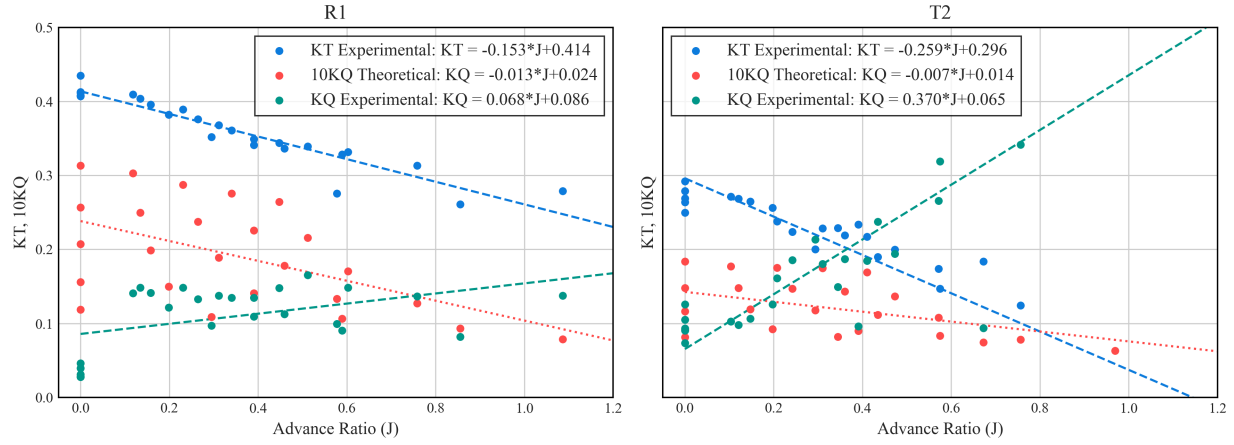


Fig. 7: Sample data and fits for propeller **R1** and **T2**, illustrating the linear fits presented in Table IV, for *all* propellers characterized for dynamic thrust. General trends are captured with linear fits of the form $K_i = K_{i,m} \cdot J + K_{i,b}$.

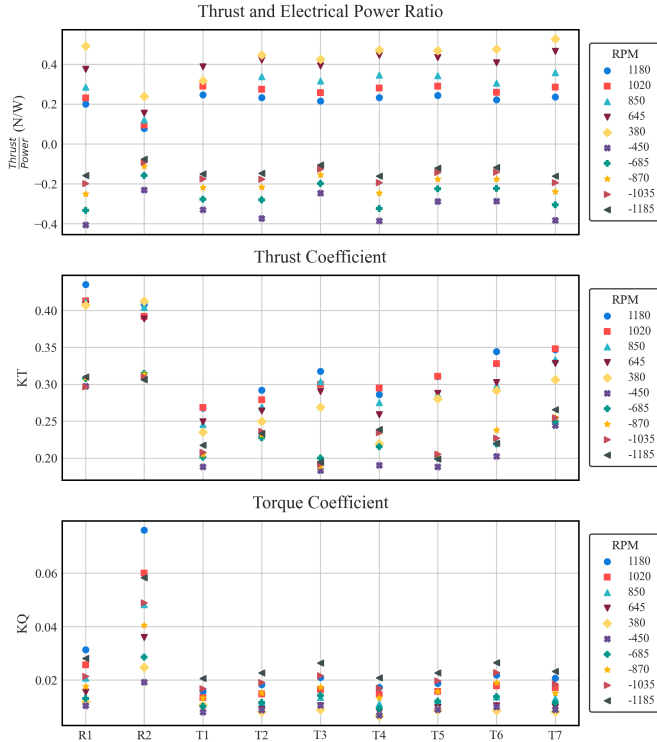


Fig. 8: Performance metrics for the propellers discussed herein. Here we present a thrust to power ratio, and the thrust and torque coefficients, both K_T and K_Q respectively.

where Ω is the rotational speed, was attempted for each propeller. Fig. 10 displays the fits for each of the propellers with the shaded region denoting the 95% confidence interval for each of the fits. For the RPM range we tested, there seems to be no significant difference in noise levels between the three propellers that were tested because many of the shaded intervals overlap, indicating that with 95% confidence, there is no statistical significance in acoustic performance between

Table IV: Linear fit coefficients and RMSE values for thrust (K_T) and torque (K_Q) coefficients of various propellers. Propeller names in bold are depicted fully in Fig. 7.

Prop.	$K_{T,m}$	$K_{T,b}$	K_T RMSE	$K_{Q,m}$	$K_{Q,b}$	K_Q RMSE
R1	-0.1528	0.4137	0.0153	-0.0134	0.0238	0.0057
T1	-0.1180	0.2507	0.0119	-0.0064	0.0127	0.0023
T2	-0.2590	0.2961	0.0308	-0.0067	0.0143	0.0032
T3	-0.1175	0.2705	0.0555	-0.0086	0.0162	0.0036
T4	-0.1250	0.2658	0.0237	-0.0081	0.0133	0.0032
T5	-0.1510	0.2954	0.0141	-0.0080	0.0148	0.0030
T7	-0.1503	0.3422	0.0166	-0.0092	0.0164	0.0040

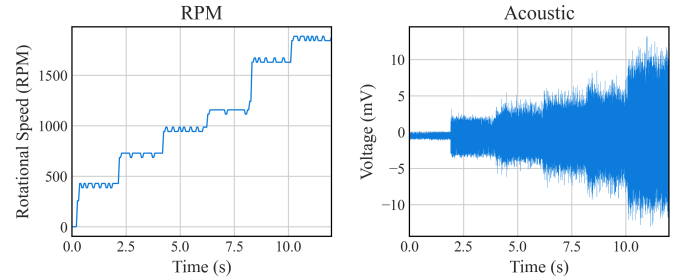


Fig. 9: A plot of raw RPM data and raw noise data from propeller R1 trial, intervals of discrete step-ups can be clearly seen in both plots indicating a step increase in PWM signals.

these propellers.

V. CONCLUSIONS AND RECOMMENDATIONS

In this work we experimentally characterized regular propellers designed in OpenProp and toroidal propellers based on the parametrization defined in [7]. We presented results from 300 trials, on two regular propellers, and seven toroidal propellers. We demonstrated how overall, the OpenProp 3-blade propeller has the highest thrust-to-electrical power ratio

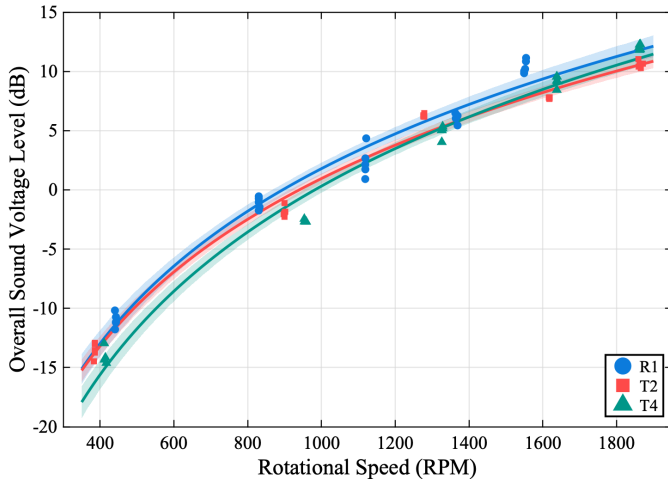


Fig. 10: The OASVL vs rotational speed of propeller was fitted with a logarithmic fit with the form $OASVL = A \ln(\Omega) + B$ and the 95% confidence interval of each fit is shaded. The R1 propeller fit parameters were $A = 16.1 \pm 1.1$ and $B = -109.4 \pm 7.8$, the T2 fit parameters were $A = 15.43 \pm 0.74$ and $B = -105.7 \pm 5.1$, and the T4 propeller fit parameters were $A = 17.4 \pm 1.0$ and $B = -119.7 \pm 7.2$.

and thrust coefficient for static thrust, with slightly more torque than the toroidal propellers characterized herein. For dynamic thrust, we determine that the OpenProp **R1** propeller also has the best trend for a thrust coefficient. Lastly, during acoustic characterization, we find that among the propellers tested, the difference in acoustic performance within the operating regime tested and at this size, was not statistically significant.

This work provides a foundational dataset on this parametric representation. The authors do not try to conclude or assert strict advantages over one propeller kind or another, as the paradigms for propeller generation were fundamentally different between the OpenProp and toroidal propellers. OpenProp was used to design an optimized propeller around desired specifications, whereas the toroidal propellers used in this study were generated by adjusting nominal parameters on specific variables.

After conducting these experiments, the authors offer several recommendations for those who wish to expand on the work presented in this study. Firstly, a fundamental limitation in this study lies in the uncertainty associated with measuring torque along the x-axis using the load cell, resulting in noisy and unreliable K_Q measurements. Thrust Characterization could be improved by using a motor with more well defined specifications, thereby enabling more accurate torque measurement. Moreover, the experiments were also constrained by the maximum force capacity of the load cell and the torque output of the motor. To enable a better understanding of acoustic performance across a wider range of application, this experiment should be scaled up so that larger propellers can be driven at higher RPMs to approach a cavitation regime. Finally, it should be noted that this work explores *one* specific parameterization

of a toroidal propeller, leaving much of the toroidal propeller design space unexplored. Future investigations into alternative parameterizations or novel toroidal geometries may reveal designs that outperform conventional propellers in specific applications or key performance metrics.

ACKNOWLEDGMENT

The authors would like to thank Professor David Barrett for his recent advancements on the tow-tank, and providing us with his time in training us best practices for operation. The authors would also like to thank Professor Michael Triantafyllou and MIT Sea Grant for supporting this work by providing access to the tow tank. We would also like to thank Michael Sacarny for lending us his experimental apparatus and the operational software for acoustic characterization.

REFERENCES

- [1] W. G. E. Neal, "The marine engineer, a monthly journal of marine engineering, shipbuilding, steam navigation, and electrical engineering," *The Marine Engineer*, vol. XIII, p. 241, 1892. Retrieved May 8, 2023.
- [2] F. Honerkamp, "Fan," Feb. 1942. Filed October 18, 1939.
- [3] R. L. Marlet, "Nouvelle hélice pour paquebots à grande vitesse et pour avions et aéronefs," 1892. Patent FR808801A. Retrieved May 8, 2023.
- [4] Sharrow Marine, "Sharrow marine." <https://sharrowmarine.com/>. Accessed: 2024-05-16.
- [5] MIT Lincoln Laboratory, "Technology highlight: Toroidal propeller," Tech. Rep. TVO.41, Massachusetts Institute of Technology, 2022. Approved for public release; distribution is unlimited. This material is based upon work supported by the United States Air Force under Air Force Contract No. FA8702-15-D-0001.
- [6] C. Peng, X. Zhao, and G. Liu, "Noise in the sea and its impacts on marine organisms," *International journal of environmental research and public health*, vol. 12, no. 10, pp. 12304–12323, 2015.
- [7] Y. Liyu, W. Chao, S. Cong, and G. Chunyu, "Mathematical expression method for geometric shape of toroidal propeller," *Chinese Journal of Ship Research*, vol. 19, no. 3, pp. 224–233, 2024.
- [8] B. Epps, J. Chalfant, R. Kimball, A. Techet, K. Flood, and C. Chrysostomidis, "Openprop: An open-source parametric design and analysis tool for propellers," in *Proceedings of the 2009 grand challenges in modeling & simulation conference*, pp. 104–111, 2009.
- [9] T. I. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control*. John Wiley & Sons, 2011.
- [10] J. Lam, A. Chen, A. Bennett, and M. Triantafyllou, "Propeller characterization testing of a blue robotics t200 thruster," in *OCEANS 2023 - Limerick*, pp. 1–8, 2023.
- [11] "Acoustic noise requirement," Tech. Rep. PD-ED-1259, Jet Propulsion Laboratory, National Aeronautics and Space Administration, Pasadena, CA, 1996.
- [12] R. Turrisi, H. Huang, F. Strømstad, J. Morrison, A. Papalia, J. Leonard, and M. Benjamin, "The spurdog auv: A field configurable and optionally a-sized low-cost auv," in *2024 IEEE/OES Autonomous Underwater Vehicles Symposium (AUV)*, pp. 1–6, 2024.
- [13] R. Turrisi, J. Beason, and M. Zhou, "Towards the development of the multipurpose autonomous underwater vehicle," in *Global Oceans 2020: Singapore – U.S. Gulf Coast*, pp. 1–6, 2020.

Table V: Geometric parameters of toroidal propeller

l/L	r_l/R	b/D	P/D	t/b	f/b	x_l/D	θ_s/rad	φ/rad	ψ/rad	α/rad
0.0000	0.2000	0.1613	0.8747	0.6014	0.0140	-0.0413	0.0000	-0.4810	0.0000	0.0000
0.0624	0.3000	0.1908	0.8874	0.4866	0.0304	-0.0548	-0.0155	-0.4454	0.0426	0.0021
0.1203	0.4000	0.2133	0.8910	0.3934	0.0440	-0.0622	-0.0192	-0.4091	0.0590	0.0108
0.1773	0.5000	0.2297	0.8804	0.3152	0.0550	-0.0645	-0.0157	-0.3711	0.0744	0.0251
0.2343	0.6000	0.2396	0.8538	0.2506	0.0634	-0.0619	-0.0063	-0.3304	0.0949	0.0456
0.2937	0.7000	0.2408	0.7987	0.1972	0.0689	-0.0538	0.0075	-0.2850	0.1450	0.0763
0.3590	0.8000	0.2244	0.7189	0.1548	0.0698	-0.0391	0.0255	-0.2314	0.2560	0.1203
0.4377	0.9000	0.1897	0.6211	0.1400	0.0619	-0.0170	0.0494	-0.1628	0.5023	0.1639
0.4889	0.9500	0.1637	0.5754	0.1360	0.0518	-0.0015	0.0660	-0.1143	0.7475	0.1824
0.5250	0.9750	0.1493	0.5533	0.1340	0.0431	0.0093	0.0777	-0.0796	0.9678	0.1916
0.5729	0.9950	0.1369	0.5377	0.1320	0.0298	0.0231	0.0932	-0.0325	1.3249	0.1991
0.6000	1.0000	0.1347	0.5373	0.1340	0.0216	0.0305	0.1025	-0.0042	1.5708	0.2009
0.6438	0.9950	0.1420	0.5685	0.1360	0.0083	0.0410	0.1168	0.0428	2.0605	0.1969
0.6842	0.9750	0.1587	0.6341	0.1400	-0.0035	0.0488	0.1299	0.0871	2.4330	0.1836
0.7146	0.9500	0.1748	0.6933	0.1454	-0.0113	0.0526	0.1395	0.1213	2.6485	0.1566
0.7574	0.9000	0.2019	0.7886	0.1734	-0.0203	0.0546	0.1527	0.1707	2.8627	0.0967
0.8150	0.8000	0.2351	0.9306	0.2276	-0.0268	0.0482	0.1696	0.2403	3.0280	0.0407
0.8582	0.7000	0.2492	1.0289	0.2820	-0.0276	0.0358	0.1815	0.2937	3.0836	0.0251
0.8941	0.6000	0.2483	1.0911	0.3372	-0.0260	0.0199	0.1909	0.3393	3.1008	0.0171
0.9249	0.5000	0.2360	1.1124	0.3932	-0.0230	0.0016	0.1988	0.3794	3.1154	0.0110
0.9523	0.4000	0.2168	1.1091	0.4492	-0.0190	-0.0181	0.2054	0.4157	3.1264	0.0066
0.9771	0.3000	0.1929	1.1016	0.5052	-0.0147	-0.0389	0.2114	0.4492	3.1350	0.0030
1.0000	0.2000	0.1653	1.0873	0.5614	-0.0100	-0.0607	0.2166	0.4810	3.1416	0.0000